

NEUTRINO MASS FROM SZ SURVEYS

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The expected sensitivity of cluster SZ number counts to neutrino mass in the sub-eV range is assessed. We find that from the ongoing *Planck*/SZ measurements the (total) neutrino mass can be determined at a (1σ) precision of 0.06 eV, if the mass is in the range 0.1 – 0.3 eV, and the survey detection limit is set at the 5σ significance level. The mass uncertainty is predicted to be lower by a factor $\sim 2/3$, if a similar survey is conducted by a cosmic-variance-limited experiment, a level comparable to that projected if CMB lensing extraction is accomplished with the same experiment. At present, the main uncertainty in modeling cluster statistical measures reflects the difficulty in determining the mass function at the high-mass end.

1. Introduction

If the sum, m_ν , of all (three) neutrino masses is close to ~ 0.3 eV, and thus comparable to the energy scale of the recombination epoch, then the earliest measurable effect of massive neutrinos on the CMB is their impact on the early integrated Sachs Wolfe (ISW) effect. With a mass lower than this value neutrinos were a relativistic component that contributed to the decay of linear gravitational potentials and thus caused a net change in the temperature of the CMB. Measurements of CMB polarization can also constrain neutrino masses from measurements of the B-mode lensing-induced signal by the large scale structure at redshifts of a few. Ongoing ground-based CMB experiments have been searching for this signal at its expected peak $l \sim 1000$, on smaller angular scales than the predicted, much weaker primordial (inflation-induced) B-mode signal which is expected to peak around $l \sim 100$.

It has been conjectured that employing optimal estimators to CMB temperature and polarization maps obtained from full-sky measurements with a cosmic-variance-limited (CVL) experiment will allow recovering the lensing potential with precision that could constrain m_ν at a level of ~ 0.04 eV.¹ This projection was based on the assumption that foregrounds are negligible and there is no source of non-gaussianity other than CMB lensing, and therefore it is likely to be overly optimistic. Analysis of the WMAP7 database (with BAO and H_0 priors) yielded the upper limit² $m_\nu < 0.58$ eV (95% CL). Other cosmological probes of neutrino masses include weak lensing shear maps,^{3–5} galaxy,^{6,7} and Ly α surveys.^{8,9}

To assess the relative importance of the yields of these various cosmological probes, their respective upper limits have to be compared with lower limits from neutrino oscillation experiments. Measured mass values from neutrino oscillations imply that at least one of the neutrinos is 0.05 eV or heavier, if the mass hierarchy is 'normal', whereas in the 'inverted' hierarchy two neutrino masses are each above 0.05 eV. Thus, the lowest bound on m_ν is in the range 0.05-0.10 eV, which sets the benchmark for determining the hierarchy and possibly ruling out one of these hierarchies. This sets the goal for cosmological neutrino mass precision to better than 0.05 eV.

The SZ effect is a unique probe of cluster and cosmological parameters; its statistical diagnostic value is gauged by cluster number counts and the power spectrum of the CMB anisotropy it induces. The steep dependence of SZ number counts on $\sigma(M, z)$ - the rms mass fluctuation on mass scale M at redshift z - which depends exponentially on m_ν (e.g., Ref. 10), makes cluster surveys sensitive probes of neutrino mass, as has already been demonstrated.¹¹⁻¹³

In this paper we summarize results from our recent analysis¹⁴ in which we extended and improved our earlier predictions¹² of the precision with which m_ν can be determined from SZ measurements by the ongoing *Planck* and future CVL surveys. Our approach and more details of the Fisher matrix analysis are only very briefly discussed here; a more detailed description is given in Refs. 12,14.

2. LSS Evolution with Massive Neutrinos

Evolution of the large scale structure in the matter-dominated era is described in terms of the matter power spectrum,

$$P_m(k, z) = Ak^n T^2(k, z), \quad (1)$$

where Ak^n is the primordial density fluctuation spectrum with index n , and normalization A ; $T(k, z) = T(m_\nu; k, z)$ is the transfer function. Normalization of the power spectrum measured at present is in terms of the mass variance parameter on a scale of $R = 8 \text{ Mpc h}^{-1}$,

$$\sigma_8^2 = \int_0^\infty P_m(k, z) W^2(kR) k^2 \frac{dk}{2\pi^2}, \quad (2)$$

where $W(kR)$ is a window function (typically assumed to be top-hat). The essence of neutrino impact on density fluctuations can be appreciated from the fact that for masses $\leq 1 \text{ eV}$, diffusion damping of density fluctuations is on scales below a few tens of Mpc. The suppression of these scales is represented in the transfer function. It is expected that m_ν and σ_8 are anti-correlated, and due to the steep dependence of cluster counts on σ_8 , we expect also strong dependence on m_ν .

The total neutrino mass can be deduced from a comparison of the observed number of clusters in a given redshift bin to the number predicted from the mass function, $\frac{dn(M; z)}{dM}$, which is defined in terms of the differential number of clusters in

a volume element dV ,

$$dN(M, z) = f_{sky} \frac{dn(M, z)}{dM} dV dM, \quad (3)$$

where f_{sky} is the observed sky fraction (0.65 for the two experiments considered here). The number of clusters in a given interval Δz around z_i is

$$\Delta N(z_i) = f_{sky} \Delta z_i \frac{dV(z_i)}{dz} \int \frac{dn(M, z_i)}{dM} dM. \quad (4)$$

In the analysis described here we used the code in Ref. 15 to calculate $T(m_\nu; k, z)$, but we did use the default transfer function in (the CMB analysis program) CAMB to calculate the primordial angular power spectrum of the CMB. This procedure is justified given that the main difference between the two transfer functions is only apparent at large values of k .

The Tinker et al. (2008) mass function,¹⁶ which was obtained from a large set of dynamical cosmological simulations in the Λ CDM model, was adopted. With the mass function expressed in the usual form,

$$\frac{dn}{dM} = f(\sigma) \frac{\rho_m}{M} \frac{d \ln(\sigma^{-1})}{dM}, \quad (5)$$

the analytic approximation¹⁶ is

$$f(\sigma) = A \left[\left(1 + \frac{\sigma}{b} \right)^{-a} \right] e^{-\frac{c}{\sigma^2}}. \quad (6)$$

The parameters A , a , b and c , which depend on z and the overdensity at virialization, Δ_v , are

$$\begin{aligned} A &= A_0 (1+z)^{-0.14} \\ a &= a_0 (1+z)^{-0.06} \\ b &= b_0 (1+z)^{-\alpha} \\ \log(\alpha) &= - \left(\frac{0.75}{\log(\Delta_v/75)} \right)^{1.2} \\ b_0 &= 1.0 + (\log(\Delta_v) - 1.6)^{-1.5} \end{aligned} \quad (7)$$

where c and A_0 were obtained from Table 2 of Tinker et al. (2008). Values listed in the table were used also for deriving fits for a_0 and b_0 as functions of Δ_v ,

$$\begin{aligned} a_0 &= 1.7678\alpha_1 - 0.5941\alpha_2 \exp(-0.02924\Delta_v^{0.5967}) \\ c &= 1.7077\alpha_3 - 0.7038\alpha_4 \exp(-0.001\Delta_v^{1.079}) \end{aligned} \quad (8)$$

where we introduced the additional four ('nuisance') parameters $\alpha_1 - \alpha_4$ in order to account for possible uncertainties (UCs) or biases in the Tinker et al. parameters.

Cluster DM profiles were approximated by the NFW model, with a mass-concentration relation $c(M, z)$ taken from Ref. 17. Intracluster (IC) gas was assumed to be well described by a polytropic equation of state with index $\Gamma = 1.2$.

The solution of the equation of hydrostatic equilibrium for a polytropic gas inside the potential well of a DM halo is¹⁸

$$\rho(x) = \rho_0 \left[1 - \frac{B(\Gamma - 1)}{\Gamma} \left(1 - \frac{\ln(1+x)}{x} \right) \right]^{1/(\Gamma-1)} \quad (9)$$

where $x = r/r_s$, r_s is the scale factor of the NFW density profile, B is given by $B = 4\pi G \rho_s r_s^2 \mu m_p / k_B T_0$; μ is mean molecular weight, and m_p is the proton mass. More details on the IC gas model are given in Ref. 19.

The mass fraction of IC gas was assumed to have the scaling deduced in Ref. 20,

$$f_g(M, z) = \alpha_5 [0.125 + 0.037 \log_{10}(M_{500}/10^{15} M_\odot)] \quad (10)$$

with an added parameter α_5 (whose fiducial value is 1) to account for UCs or biases in this scaling relation. M_{500} is the total cluster mass within a sphere whose mean density is 500 that of the background. The z -dependence in the last equation is that of M_* , with M_* defined such that for a fixed redshift the mass fluctuation is $\sigma(M_*, z) = 1.686$.

The SZ power spectrum was normalized to the value measured²¹ by SPT, $C_l = 3.65 \mu K^2$ at $l = 3000$. This was done separately for each fiducial value of m_ν . We note that even though the shape of the SZ power spectrum depends on the fiducial neutrino mass, power levels are nearly the same for $\ell \sim 2000 - 3000$. Consequently, a similar number of clusters is expected to be detected, implying that neutrino mass UC are essentially independent on the assumed fiducial neutrino mass.

3. Likelihood Analysis

We constructed a likelihood function for cluster number counts and carried out the diagnostic analysis based on calculations of Fisher matrices for the primary CMB with and without lensing extraction, employing the standard approach (e.g., Refs. 12,22) which we do not describe here.

In the context of the flat Λ CDM model the global parameters were the normalization A , the power-law index n of the primordial scalar perturbations, density parameters of matter, Ω_m , and baryons, Ω_b , dark energy equation of state parameter, w , the Hubble parameter (scaled to 100 km/sec/Mpc) h , primordial helium abundance Y_p , optical depth to reionization τ , and the neutrino mass m_ν . Additionally, we adopted priors on the cosmological parameters obtained from the primary and lensed sky observed with *Planck* and a CVL experiment, with the corresponding Fisher matrices denoted by F^P and F^{LE} , respectively. We also used the prior $H_0 = 71.0 \pm 2.5$ km/sec/Mpc. In addition to the nine global parameters we introduced four parameters to account for possible UCs in parameter values of the Tinker et al. (2008) mass function, Eq. (8), and a parameter to account for UC in the gas mass fraction, Eq. (10).

Our likelihood function for cluster number counts is based on a Poisson distribution for the observed and expected number counts in redshift-shells (e.g., Ref. 23).

While for high cluster abundances a spatial cluster correlation term would generally be included, the selection of a high 5σ detection threshold ensures that only the most massive clusters are relevant for our analysis, effectively minimizing the impact of the relatively small correlation term, whose contribution to our results for the neutrino mass UC we estimate to be an increase of up to $\sim 15\%$. Cluster counts were calculated in redshift shells with width $\Delta z = 0.1$ up to $z = 1$. This width is larger than predicted photo- z redshift UCs which are at the $\sigma_z = 0.02(1+z)$ level; we verified that further refinement of the redshift bins did not affect the results. The choice of maximal redshift was based on the realization that the strength of the neutrino mass constraint is largely determined by low-redshift, high-mass clusters.

We calculated number counts in redshift bins rather than in redshift and mass bins since mass ‘slicing’ was found not to enhance the diagnostic power due to the requirements that each cell contains at least 20 clusters, the need to select wide mass bins to account for the large UC (of a factor of ~ 2) in mass determination, and the fact that mass and redshift of clusters that satisfy the high detection threshold are strongly correlated. Thus, cluster counts in redshift bins is obviously preferable to using the mass as an explicit parameter due to the fact that redshift is well defined and precisely measured.

Predicted precision in constraining m_ν is calculated from the Poissonian likelihood function for number counts in the i ’th redshift bin

$$\mathcal{L}_i(N_i^P; N_i^O) \propto \frac{(N_i^P)^{N_i^O} \exp(-N_i^P)}{N_i^O!}, \quad (11)$$

where N_i^P is the predicted (given a set of cosmological parameters and m_ν) and N_i^O the observed cluster number in the i ’th redshift bin. Since N_i^P is a function of several parameters λ_k , small deviations with respect to its expected fiducial value can be determined from $N_i^O \approx N_i^P + \sum_k \frac{\partial N_i^P}{\partial \lambda_k} \Delta \lambda_k$. The Fisher matrix for number counts is then calculated from the likelihood function

$$F_{jl}^N = -\frac{\partial^2 \ln \mathcal{L}}{\partial \lambda_j \partial \lambda_l} = \sum_i \frac{1}{N_i} \frac{\partial N_i}{\partial \lambda_j} \frac{\partial N_i}{\partial \lambda_l}. \quad (12)$$

The estimated UC in the parameter λ_j is the square root of the respective Fisher matrix element,

$$\Delta \lambda_j = (F_{jj}^N)^{-1/2}. \quad (13)$$

The full Fisher matrix is a sum of the number counts, F_{jl}^N , and either the primary CMB, F_{jl}^P , or the lensed CMB, F_{jl}^{LE} . For the calculation of the signal-to-noise \mathcal{S}/\mathcal{N} with which a cluster can be detected in a survey we assumed that main sources of noise are instrumental, primary CMB anisotropy, and point source contamination. The performance of optimal matched filters (as applied in Ref. 12) was assumed in order to estimate the abundance of detected clusters on a finely-sampled $M - z$ grid.

4. Projected m_ν Precision Levels

The standard Λ CDM cosmological model was assumed with WMAP-7 best-fit parameters, but no priors were set on the cosmological parameters, except for H_0 for which an UC of 2.5 km/sec/Mpc was assumed. As we specified in the previous section, the cluster population was described in terms of the Tinker et al. (2008) mass function with four additional parameters ($\alpha_1 - \alpha_4$) that gauge the robustness of σ_{m_ν} to UCs in the mass function. The calculated counts included all clusters in the mass range $3 \times 10^{13} M_\odot - 3 \times 10^{15} M_\odot$. We verified that the high S/\mathcal{N} detection threshold we set guaranteed that the detected clusters are actually much more massive than the low mass end of this range. We used the scaling relation deduced in Ref. 20 for the gas mass fraction in clusters, but parametrized it with an added multiplicative factor.

Table 1: *Planck* sky coverage and sensitivity parameters. Channel sensitivities and beam (FWHM) sizes are taken from Table 4 of Ref. 10, where sensitivities of polarization measurements (in the first seven channels) are also listed. Only the 100, 143 and 353 GHz channels were used in the computation of number counts; see the text for details.

f_{sky}	$\nu[\text{GHz}]$	$\theta_b[']$	$\Delta_T[\mu\text{K}]$
0.65	30	33	4.4
	44	23	6.5
	70	14	9.8
	100	9.5	6.8
	143	7.1	6.0
	217	5.0	13.1
	353	5.0	40.1
	545	5.0	401
	856	5.0	18300

For simulating cluster detection by *Planck* all nine frequency channels were used in the calculation of F^P and F^{LE} , whereas only the 100, 143 and 353 GHz channels were used in our calculations of the number counts. The relevant *Planck* specifications are listed in Table 1. Results for the projected neutrino mass UC from analyses of cluster number counts from the (ongoing) *Planck* survey and a similar (future) CVL survey are presented in Table 2. Listed in the table (from left to right) are the calculated UC in m_ν from the primary (P) CMB (both temperature and polarization anisotropy), lensing extraction (LE) of the CMB, primary CMB and number counts, and finally lensed CMB and number counts.

The values of the neutrino mass uncertainty listed in Table 2 clearly demonstrate that cluster number counts alone (but with priors from measurements of the primary CMB power spectrum and the HST prior on H_0) neutrino mass uncertainties may

Table 2: Statistical UC on total neutrino mass from cluster number counts obtained from the *Planck* and CVL SZ surveys; see the text for details.

Survey	$\alpha_1, \alpha_2, \alpha_3, \alpha_4$ UC [%]	σ_{m_ν} [eV] P	σ_{m_ν} [eV] LE	σ_{m_ν} [eV] P+N(z)]	σ_{m_ν} [eV] LE+N(z)]	N
<i>Planck</i>	0	0.43	0.15	0.06	0.06	6040
	3			0.07	0.06	
	5			0.08	0.07	
	10			0.12	0.09	
CVL	0	0.29	0.05	0.04	0.03	13860
	3			0.06	0.04	
	5			0.07	0.04	
	10			0.11	0.05	

be constrained to the $\sim 0.04 - 0.06$ eV range, depending on the value of m_ν and the nature of the survey.

The projected neutrino mass precision for *Planck* and a CVL experiment are based on the Tinker et al. (2008) analytic fit to their simulations. To test the robustness of our estimates to possible deviations from values of the parameters in their analytic representation, we allowed for 1 – 10% uncertainty in each of the (added) parameters $\alpha_1 - \alpha_4$ (around their fiducial value of 1). Doing so increases the neutrino mass uncertainty, σ_{m_ν} , from 0.06 eV to 0.12 when the database consists of the primary CMB and *Planck* number counts. Repeating the analysis with the primary CMB and number counts expected from a CVL survey, the corresponding σ_{m_ν} changes from 0.04 to 0.11 eV. Accounting for the inherent uncertainty in the mass function (especially at the high mass end) is clearly very important for placing reliable constraints on the neutrino mass.

5. Summary

The primary CMB anisotropy (including the lensing signature imprinted by the large scale structure) is not an optimal probe of processes and phenomena on \sim Mpc scales. Structure on \sim Mpc scales probes the entire evolutionary history of matter perturbations down to these scales. This is especially relevant to neutrino physics via the effect of neutrino free streaming on these and larger scales. Free streaming of neutrinos with masses $O(0.1)$ eV affects the matter power spectrum on the characteristic scales of galaxy clusters; this is what makes cluster number counts a more optimal probe of m_ν in this mass range.

For the observationally-allowed range $m_\nu \sim 0.1 - 0.3$ eV, the projected uncertainty in m_ν is relatively small, in the range $\sim 0.04 - 0.06$ eV, a range that is competitive with predicted results from CMB lensing extraction. We conclude that our results provide strong motivation for performing the analysis described here with actual (rather than projected) ongoing *Planck* survey data, and - if still

relevant - with future results from a CVL experiment.

As we have noted, the most important source of uncertainty in modeling the evolution of clusters in mass and redshift is the mass function. Estimated uncertainties in this basic function were explicitly included in our analysis. Extensive cosmological hydrodynamical simulations are expected to provide a more accurate description of the population, and significantly improved sampling of cluster abundance particularly at the high-mass end. Uncertainties in modeling IC gas and the evolution of the gas mass fraction are also relevant.

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